Testing the Relativistic-Microwave Theory of Ball Lightning with Plasma Simulations

Amanda Elliott

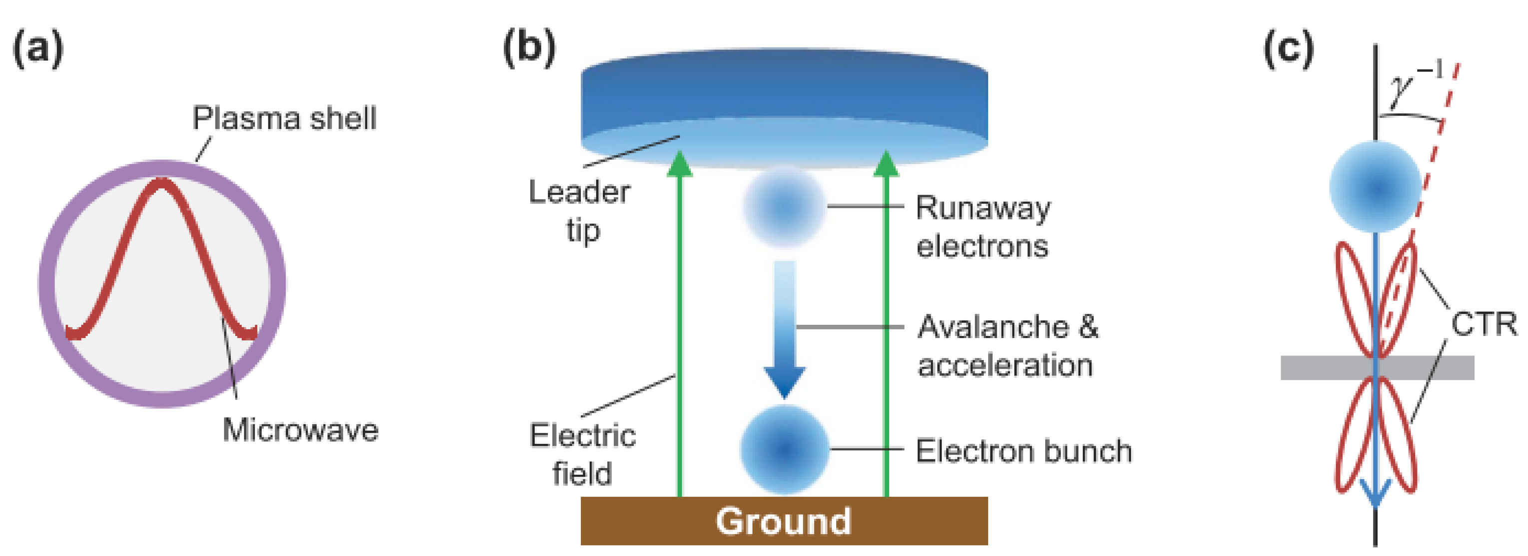
Faculty Advisor: Dr. Vladislav Bukshtynov,

Dept. of Mathematical Sciences, Florida Institute of Technology

NORTHROP GRUMMAN ENGINEERING & SCIENCE STUDENT DESIGN SHOWCASE FLORIDA TECH

Introduction

Ball lightning (BL) is an unexplained phenomenon reported by thousands of eyewitnesses as a fireball, a few cm to 1 m in diameter, moving unpredictably and independently of the wind, sometimes observed during lightning storms. Here a potential theory for the creation of BL is explored [1]. The results of this research will advance lightning protection and aviation safety.



(a). BL structure is a hollow plasma shell that contains a **standing microwave** energy source. This model explains the two most common types of BL termination: silent dissipation like a gas, and a violent, potentially lethal, explosion.

Steps in BL creation:

1. (b). In the last leader step of a lightning strike, a **bunch of runaway electrons** emerge from the leader tip. The bunch is accelerated to a relativistic speed by the electric field between the leader and ground and undergoes an avalanche.
2. (c). **Coherent Transition Radiation (CTR)** is produced by the electron bunch striking the ground or passing through conductors.
3. This radiation ionizes the air, trapping the wave in a **plasma shell**.

Mathematical Model

Unitless Maxwell-Lorentz equations [2]:

$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$
$$\nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + 2\pi \boldsymbol{J}$$
$$\nabla \cdot \boldsymbol{E} = 2\pi\rho$$
$$\boldsymbol{F}_L = \frac{dP}{dt} = \frac{2\pi q}{M}(\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B})$$

Normalization laser-related units:

$$\frac{\mathbf{r}}{\lambda}, \frac{2\pi t}{\omega}, \frac{e\mathbf{E}}{m_e\omega}, \frac{\rho}{en_c}, \frac{\mathbf{J}}{en_c c}, \frac{\mathbf{P}}{Mc}, \frac{M}{m_e}, \frac{\mathbf{V}}{c}, \frac{q}{e}$$

With reference variables:

laser wavelength λ, laser angular frequency ω, critical plasma density $n_c = \varepsilon_0 m\omega^2/e^2$, electron mass m_e, mass of species M.

Computational Methods

An open-source Particle-in-Cell (PIC) code called Smilei [3] is used to simulate two parts of the theory.

Vlasov-Maxwell Collisionless Plasma

Both species, ions and electrons, are described by a distribution function $f_s(t, \boldsymbol{x}, \boldsymbol{p})$, satisfying

$$\left(\partial_t + \frac{\boldsymbol{p}}{m_s\gamma} \cdot \nabla + \boldsymbol{F}_L \cdot \nabla_p\right) f_s = 0$$

with the relativistic Lorentz factor $\gamma = \sqrt{1 + \boldsymbol{p}^2/{m_s}^2}$

Relativistic Equations of Motion

$$d\boldsymbol{x}_p/dt = \boldsymbol{u}_p/\gamma$$
$$d\boldsymbol{u}_p/dt = \frac{q_s}{m_s}\left(\boldsymbol{E}_p + (\boldsymbol{u}_p/\gamma) \times \boldsymbol{B}_p\right) - \boldsymbol{F}_{pond}$$

Pondermotive force: $\boldsymbol{F}_{pond} \propto \nabla \boldsymbol{E}^2$

Assumptions and Simplifications

Ionization is not simulated. Particles are initialized with a cold momentum distribution, $T_e = 0$ eV. The mass of ions is the average of air molecules ~28.97 g/mol. The conductor along the $x = 0$ boundary is simulated as an overdense plasma ($n_0 \geq n_c$) with profile: $n_e = n_i = n_0 \exp(-x^2/\sigma^2)$, with $\sigma = 4$ cm.

References

[1] H-C. Wu. Relativistic-Microwave Theory of Ball Lightning. Scientific Reports 6, 28263, 2016.

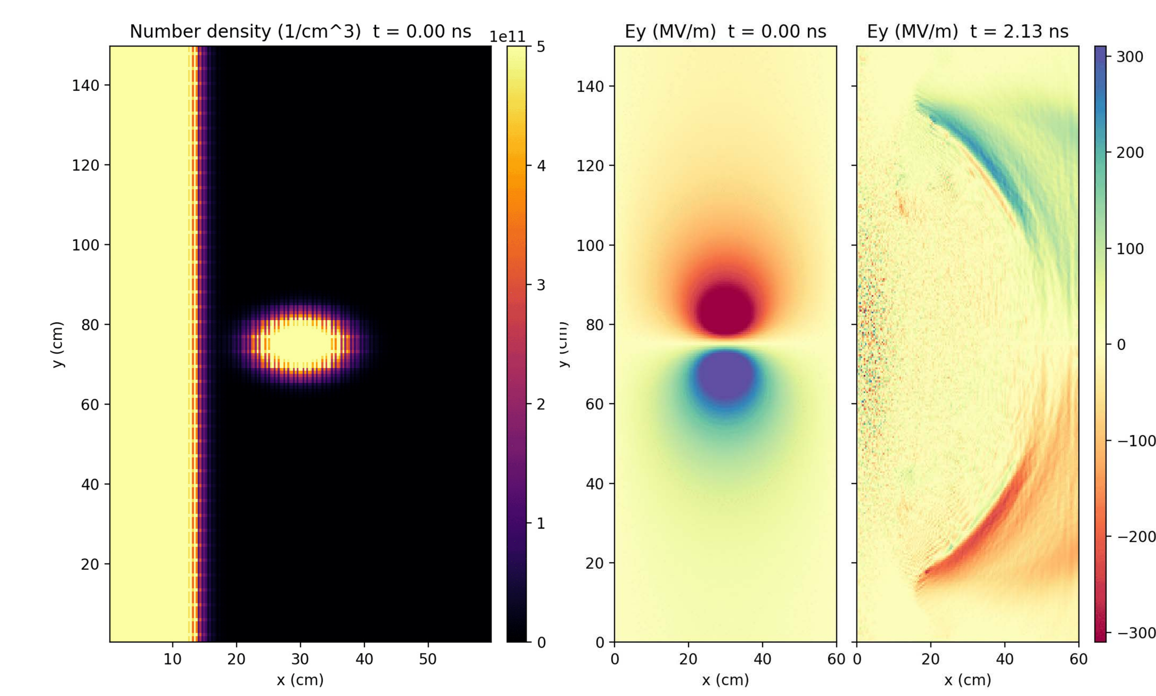
[2] H-C. Wu. JPIC & How to make a PIC code. 2011.

[3] Smilei: smileipic.github.io/Smilei/index.html

Results

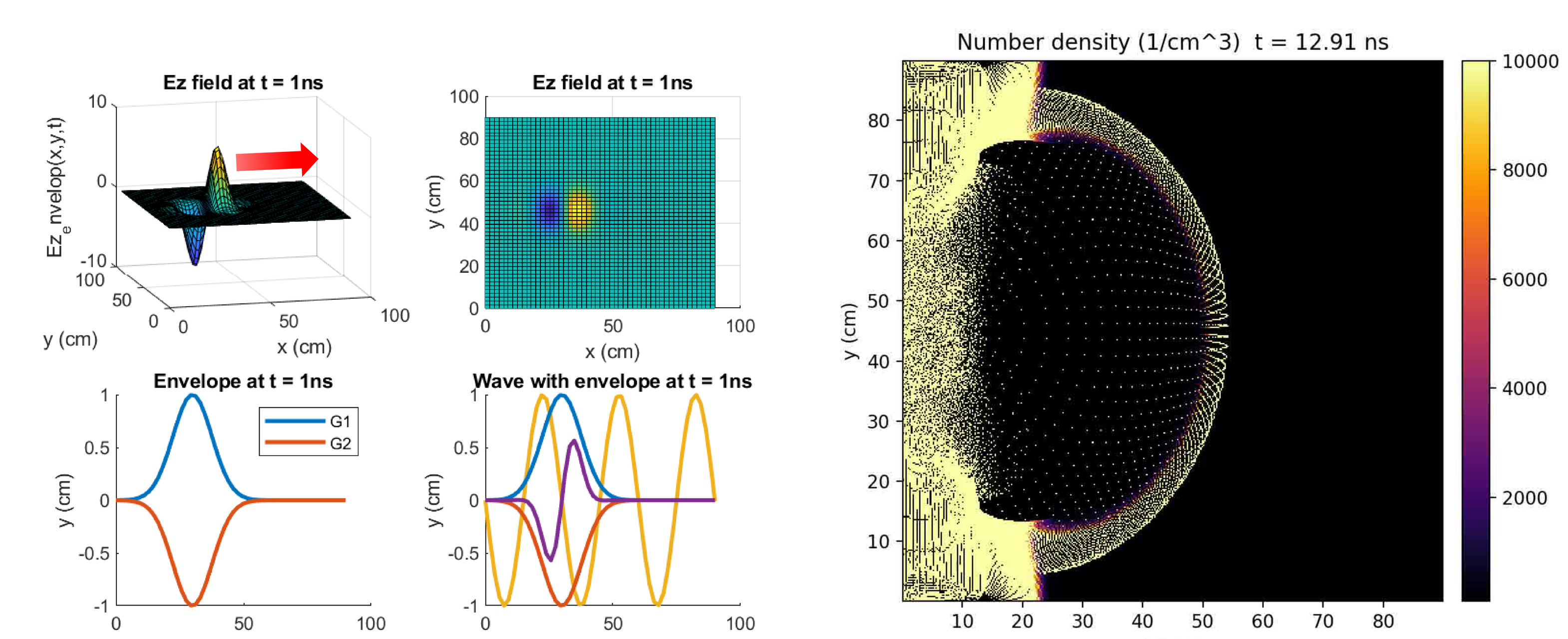
Simulation 1: Microwave Pulse Generation

A relativistic (50 MeV) electron bunch with profile $n_b = n_{b0} \exp(-R^2/\sigma^2)$ and $n_{b0} = 3.7 \cdot 10^{11}\ \text{cm}^{-3}$ is accelerated toward a ($n_0 = 50n_c$) conductor. A radial CTR pulse with peak $\mathrm{E_y} = 317$ MV/m is generated.



Simulation 2: Microwave Bubble Trapping

Input: $E_z = \exp[-(y/R)^2] \cdot G_1 \cdot G_2 \cdot \sin(\omega(t - x/c))$ with $G_1 = \exp[-[(t - x/c)/wR]^2]$, $G_2 = -G_1$. $w = 0.01\,\tau$ for a $\tau = 2$ ns duration. Plasma shell forms in conductor ($n_0 = 4n_c$) but expands without stabilizing.



Conclusions

Microwave pulse generation based on previous results [1] was replicated in Smilei [3], while the bubble trapping was not. The original results were generated by the author's personal PIC code [2] and may not be reproducible on standard PIC codes that are designed for vacuum. Atmospheric molecule collisions with electrons cannot be simulated at this time scale ($10^{12} \gg 10^9$ Hz), but the resulting energy loss can be approximated [1] and may help stabilize the plasma shell.

Acknowledgements

Dr. Jean C. Perez for his stellar plasma physics education.