Fig: [Boerner, 2019]

# Testing the Relativistic-Microwave Theory of Ball Lightning using Plasma Simulations

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Physics

# Agenda

## What is Ball Lightning (BL)?

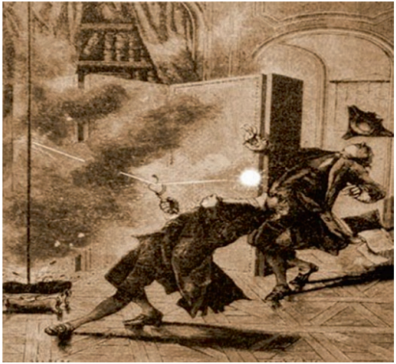


Fig: Engraving of the ball lightning caused death of Russian physicist Georg Wilhelm Richmann in 1753 [Boerner, 2019].

- Rare and unexplained weather phenomenon that has consistent behavior attested by thousands of eyewitness reports.
- Observers report seeing a ball of light, about 20-50cm in diameter, moving horizontally, often against the wind.
- Typical BL observations are short and may last less than a minute.
- Reports are often associated with nearby lightning strikes, but the exact mechanisms that create BL objects are unknown.
- Investigated by Faraday, Kelvin, Arrhenius, Boyle, and Arago.

# Why Study Ball Lightning

Fig: [Boerner, 2019]

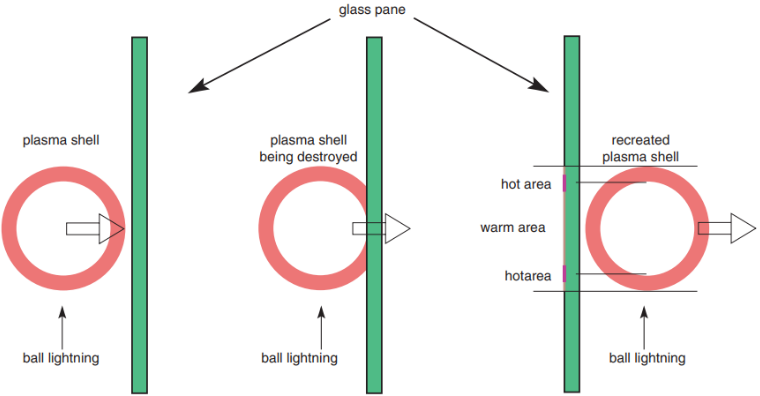
- Expansions of knowledge in fields such as plasma physics and atmospheric electricity.
- Aviation Safety: BL near planes has interfered with navigation equipment and has been known to enter the plane through the cockpit.
- Discovering how BL maintains its stability while in a spherical geometry could potentially lead to improvements in plasma confinement methods, which presently requires powerful magnetic fields to be contained.

## Properties of Ball Lightning (1/2)

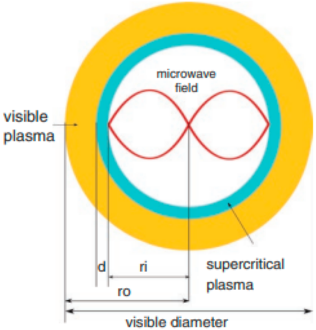
- Spherical shape reported in 90% of cases.
- Diameter: a few centimeters to a meter.
- Lifespan: A second to a few minutes.
- Color: Ranging from deep red, through orange-yellow to blue and blinding white. Only green is rarely seen.
- Luminosity: BL objects emit energy, so they will store a certain amount of energy.
- Creation:
  - Often BL is preceded by an initial flash of a linear lightning strike, near which BL is formed.
  - The existence and creation of BL objects in closed rooms and in modern, all-metal aircraft have often been reported.
- Horizontal movement independent of the wind. Hovers about 1 m above the ground and moves at 2 m/s.

## Properties of Ball Lightning (2/2)

- Termination: Either dissipates silently like a gas or violently with an explosive sound and burst of energy.
- Passage Through Objects: BL objects have been observed passing through holes, curtains, and windows; sometimes even through metal screens. Fig: [Boerner, 2019].



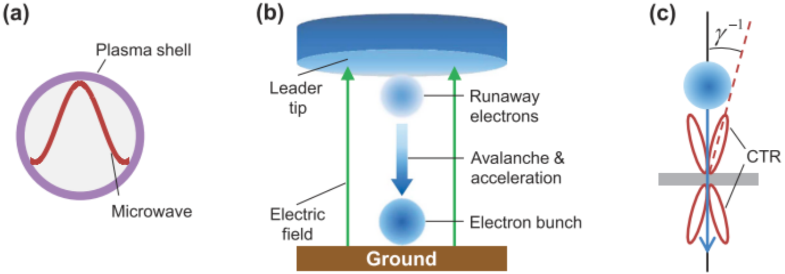
## Wu's Relativistic Microwave Theory (1/2)




- The BL object is separated into two sections, a plasma outer shell and a core, which may be a vacuum.
  Fig: [Boerner, 2019].
- The plasma shell would provide the necessary structure to contain a standing microwave.

- The microwave is the energy source that could transverse windows. The energy transmitted through the window would then re-ionize the air on the opposite side.
- Microwaves would also explain the most commonly reported diameters for these objects (a few cm to 1 m) which coincides with the wavelength range of microwaves, 1 mm to 1 m.

# Wu's Relativistic Microwave Theory (2/2) [Wu, 2016]




1. In the last leader step of a lightning strike, a bunch of runaway electrons emerges from the leader tip, is accelerated to a relativistic speed by the electric field between the leader and ground and undergoes an avalanche.
2. Coherent transition radiation (CTR) is produced by the electron bunch striking the ground or passing through aircraft skins.
3. This intense (310 MV/m) wave is trapped in a plasma shell.

## Radiation Pressure / Ponderomotive Force

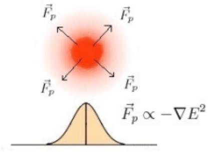



Fig: [E. Mouziouras. Comparison of EPOCH and SMILEI (2019)]

Important parameter for electric field (ie: laser) and plasma interactions.
Critical number density of plasma: $n_c = \epsilon_0 m\omega^2/e^2$

$n << n_c$: Laser passes through plasma in complete transmission.
$n >> n_c$: Laser is reflected by the plasma.

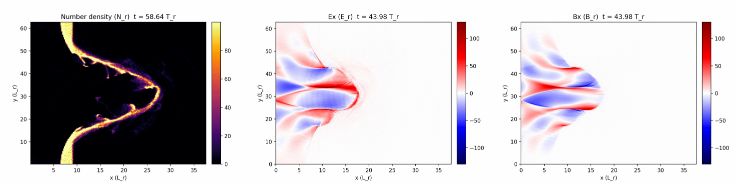



Fig: Column of plasma with $n = 100n_c$, and a laser incident from the left.

## Wu's Mathematical Model

Normalizations:

$$\frac{\mathbf{r}}{\lambda}, \frac{2\pi t}{\omega}, \frac{e\mathbf{E}}{m_e \omega c}, \frac{e\mathbf{B}}{m_e \omega}, \frac{\rho}{e n_c}, \frac{\mathbf{J}}{e n_c c}, \frac{\mathbf{P}}{Mc}, \frac{M}{m_e}, \frac{\mathbf{V}}{c}, \frac{q}{e},$$

where $\lambda$ is the laser wavelength in vacuum, $\omega$ is the laser angular frequency, $n_c$ is the critical plasma density, $m_e$ is the electron mass, $e$ is the fundamental charge, $c$ is the light speed in vacuum, and $M$ can be the electron or ion mass.

Maxwell equations before and after normalization:

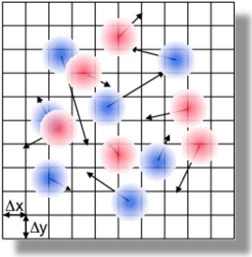
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

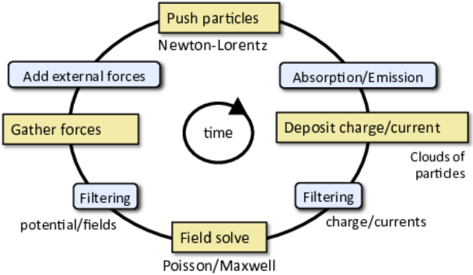
$$\nabla \times \mathbf{B} = \mu_o \epsilon_o \frac{\partial \mathbf{E}}{\partial t} + \mu_o \mathbf{J} \qquad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + 2\pi \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_o \qquad \nabla \cdot \mathbf{E} = 2\pi \rho$$

$$\frac{d\mathbf{P}}{dt} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \qquad \frac{d\mathbf{P}}{dt} = \frac{2\pi q}{M}(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$

## Computational Methods: The Particle-in-Cell (PIC) Loop




During initialization, all particles in the simulation have been loaded and the EM fields have been computed over the simulation grid. Fig: [WarpX PIC Theory]. Each time step consists of:

1. interpolating the electromagnetic fields at the particle positions,
2. computing the new particle velocities and positions,
3. projecting the new charge and current densities on the grid,
4. computing the new electromagnetic fields on the grid.

## Comparison of Wu and Smilei Models (1/2)

Smilei is an open source PIC code:
*https://smileipic.github.io/Smilei/index.html*

Vlasov-Maxwell Model: Kinetic description of collisionless plasma. Each species is described by their respective distribution functions $f_s(t, \mathbf{x}, \mathbf{p})$, where $s$ denotes a given species consisting of particles of charge $q_s$, and mass $m_s$

$$\left(\partial_t + \frac{\mathbf{p}}{m_s \gamma} \cdot \nabla + \mathbf{F}_L \cdot \nabla_{\mathbf{p}}\right) f_s = 0,$$

where $\gamma = \sqrt{1 + \mathbf{p}^2/m_s^2}$ is the relativistic Lorentz factor, and the Lorentz force is

$$\mathbf{F}_L = q_s(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

Maxwell equations solved numerically using the Finite Difference Time Domain approach.

# Comparison of Wu and Smilei Models (2/2)

## Relativistic Equations of Motion

$$\frac{d\mathbf{x}_p}{dt} = \frac{\mathbf{u}_p}{\gamma_p}$$

$$\frac{d\mathbf{u}_p}{dt} = r_s\left(\mathbf{E}_p + \frac{\mathbf{u}_p}{\gamma_p} \times \mathbf{B}_p\right) - r_s^2 \frac{1}{4\gamma_p}\nabla(|\tilde{\mathbf{A}}_p|^2)$$

where $r_s = q_s/m_s$ is the charge-over-mass ratio for species $s$, and $\mathbf{u}_p = \mathbf{p}_p/m_s$ is the reduced momentum. The particle's momentum $\mathbf{p}$ and position $\mathbf{x}$ are computed using a second-order leap-frog integrator:

$$\mathbf{x}_p^{n+1} = \mathbf{x}_p^n + \Delta t \frac{\mathbf{u}_p^{n+\frac{1}{2}}}{\gamma_p}$$

$$\mathbf{u}_p^{n+\frac{1}{2}} = \mathbf{v}_p^{n-\frac{1}{2}} + r_s \Delta t \left[ E_p^{(n)} + \frac{\mathbf{v}_p^{(n+\frac{1}{2})} + \mathbf{v}_p^{(n-\frac{1}{2})}}{2} \times B_p^{(n)} \right]$$

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# Simulation 1: Microwave Pulse Generation (1/2) Desired Result [Wu, 2016]

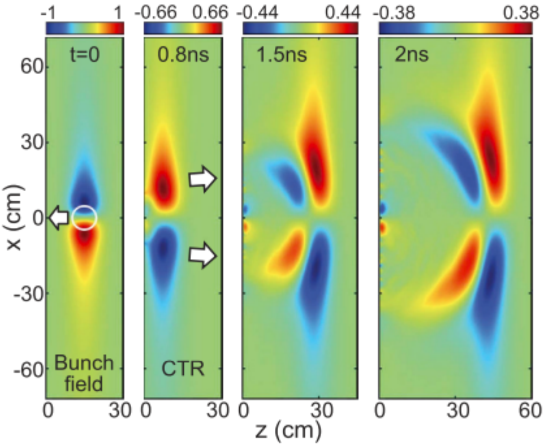


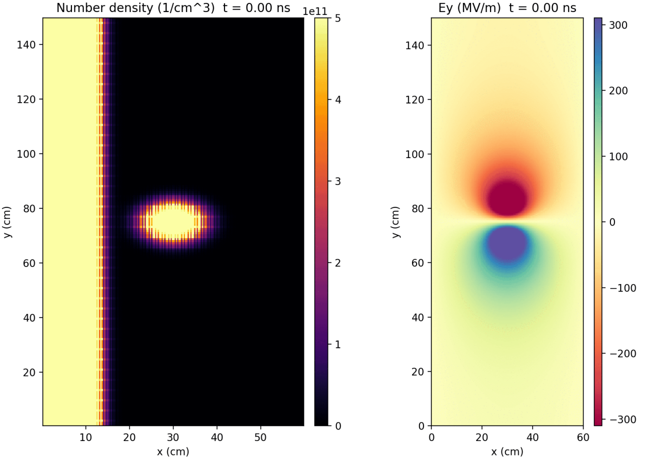

Fig: Electric field with peak amplitude ≈ 310 MV/m.

## Inputs

- Conductor at $z = 0$ as over-dense, cold, neutral plasma with density $n_0 e^{-z^2/2\sigma^2}$
  - $n_0 = 50 n_c$
  - $n_c = \epsilon_0 m \omega^2 / e^2$
  - $\sigma = 4\text{cm}$
- Electron bunch with energy of 50 MeV, speed 0.99c, and density $n_{b0} exp[-(x^2 + z^2)/(2\sigma^2)]$ and $n_{b0} = 3.7 \cdot 10^{11}\ cm^{-3}$

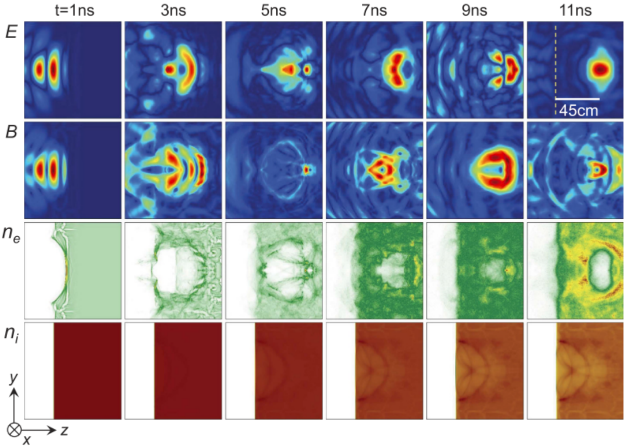
Open Boundary Conditions → remove particles
→ Perfectly Matched Layers for EM.

# Simulation 1: Microwave Pulse Generation (2/2) Result




Maximum electric field magnitude $|E_y| \approx 317$ MV/m

# Simulation 2: Microwave Bubble Trapping (1/3) Desired Result [Wu, 2016]

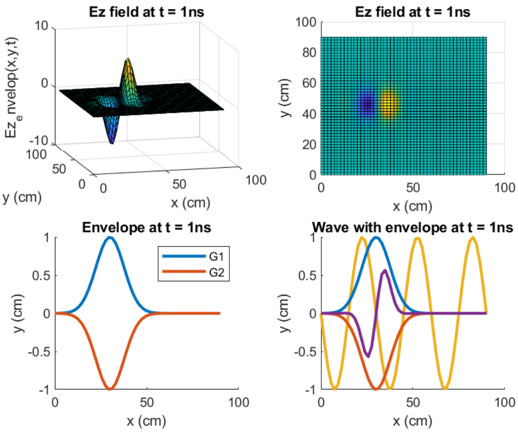



Field: $E_x = E_0 exp(-y^2/R^2) sin^2[\pi(t - z/c)/\tau] sin[\omega(t - z/c)]$,
$E_0 = 310$ MV/m, R $= 9$cm, $\tau \leq 2ns$, $\omega/2\pi = 1$ GHz

Conductor: $n_0 = 4n_c$.

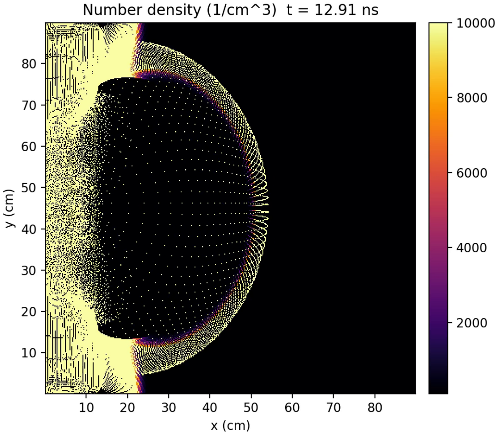
## Simulation 2: Microwave Bubble Trapping (2/3) Input Field

Amplitude: $E_0 = 310$ MV/m, Period: $\tau = 2$ ns



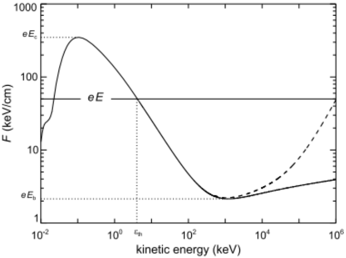

$E_z = E_0 \cdot exp[-(y/R)^2] \cdot G_1 \cdot G_2 \cdot sin[\omega(t - x/c)]$ Gaussian wave envelope:
$G_1 = exp[-[(t - x/c)/wR]^2]$,
$G_2 = -G_1$ and $w = 0.01\tau$.

# Simulation 2: Microwave Bubble Trapping (3/3) Result




- Electron number density shows spherical shell formation, but it is not stable.
- Ions within conductor are not mobile.

## Model Extensions




➜ "Simulate" Air Collisions: Approximate by energy loss. Fig: [Dwyer, 2012].

- Introduce Vertical Forces: Gravity, the upward convective force, and the mirror force with the ground.
- Quantify Stability: Monitor internal pressure balance and plasma shell surface tension.
- Extend Lifespan: Explore the role of humidity and moisture.
- Simulate Window Permeation: Internal standing wave should not be disturbed if the thickness of permeated material is much thinner than the microwave wavelength.
- Model BL Movement: Around obstacles and conductors. BL has been observed to navigate around objects and has inconsistent behavior when encountering conductors.

## Conclusions

- Simulation 1: The production of a high, $> 310$ MV/m, electric field by relativistic electrons accelerated toward a conductor was replicated with a field of 317 MV/m and $10^{14}$ electrons/cm$^3$. The role of electron bunch stability is unclear.
- Simulation 2: Shows promise for standing microwave in plasma. The standing wave must be trapped within two cycles, 2 ns.

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### References

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# Final Remarks

## Acknowledgments

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## More information